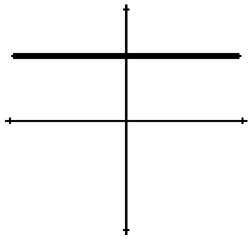
Toolkit of Functions

Students should know the basic shape of these functions and be able to graph their transformations without the assistance of a calculator.

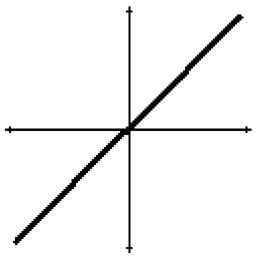
Constant

f(x) = a



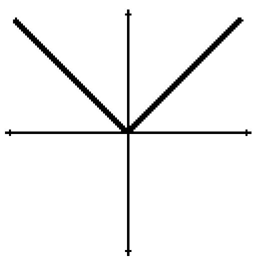
Identity

f(x) = x



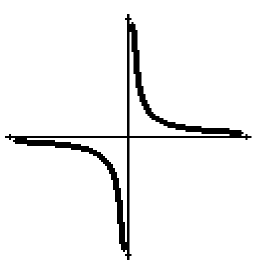
Absolute Value

f(x) = | x |



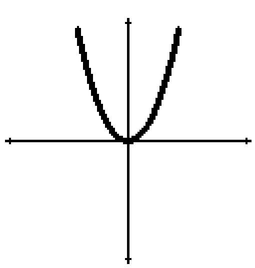
Reciprocal

$f(x) = \frac{1}{x}$



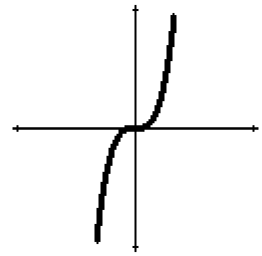
Quadratic

$f(x) = x^2$



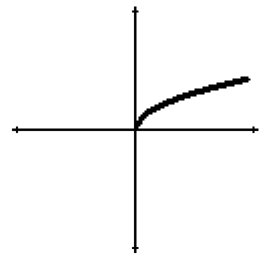
Cubic

$f(x) = x^3$



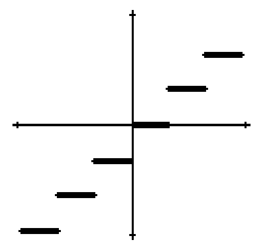
Square Root

$f(x) = \sqrt{x}$



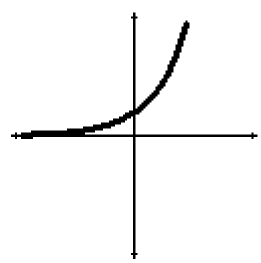
Greatest Integer

f(x) = [x]



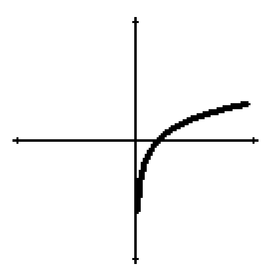
Exponential

$f(x) = a^x$



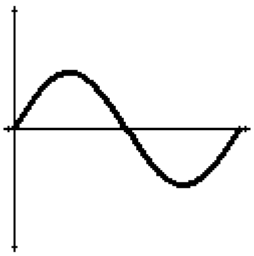
Logarithmic

$f(x) = \ln x$

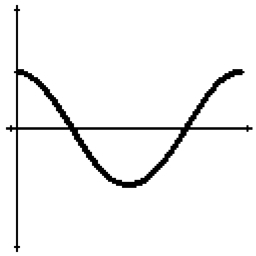


Trig Functions

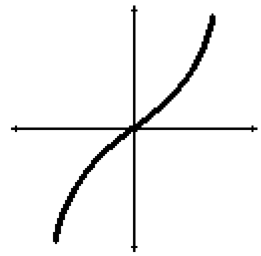
$f(x) = \sin x$



$f(x) = \cos x$



$f(x) = \tan x$



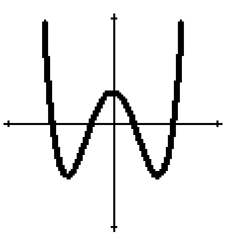
Polynomial Functions:

A function P is called a polynomial if $P(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$
Where *n* is a nonnegative integer and the numbers a_0, a_1, a_2, … a_n are constants.

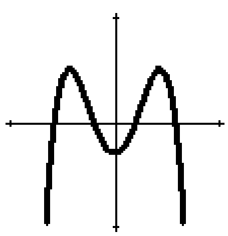
Even degree

Leading coefficient sign

Positive



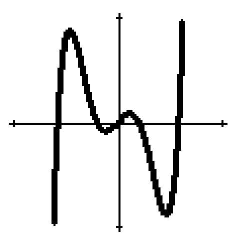
Negative



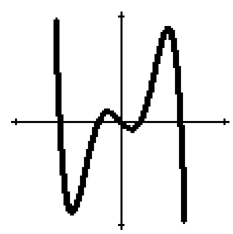
Odd degree

Leading coefficient sign

Positive



Negative



- Number of roots equals the degree of the polynomial.
- Number of x intercepts is less than or equal to the degree.
- Number of “turns” is less than or equal to (degree – 1).

Formulas and Identities

Trig Formulas:

Arc Length of a circle: $L = r\theta$ or $L = \frac{d}{360} \bullet 2\pi r$

Area of a sector of a circle: $\text{Area} = \frac{1}{2} r^2\theta$ or $\text{Area} = \frac{d}{360} \bullet \pi r^2$

Solving parts of a triangle:

Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Area of a Triangle:

$\text{Area} = \frac{1}{2} bc \sin A$ or $\text{Area} = \frac{1}{2} ac \sin B$ or $\text{Area} = \frac{1}{2} ab \sin C$

Heron's formula : $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$, where s = semi perimeter

Ambiguous Case:

θ is acute

Compute: $\text{alt} = \text{adj} \bullet \sin\theta$

opp < alt	No triangle
opp = alt	1 triangle (right)
opp>adj	1 triangle

alt<opp<adj 2 triangles

θ is obtuse or right

opp $\le$ adj	No triangle
opp > adj	1 triangle

Does a triangle exist? Yes - when

(difference of 2 sides) < (third side) < (Sum of 2 sides)

Formulas and Identities, continued

Trig Identities:

Reciprocal Identities:

$$\csc A = \frac{1}{\sin A} \qquad \sec A = \frac{1}{\cos A} \qquad \cot A = \frac{1}{\tan A}$$

Quotient Identities: $\tan A = \frac{\sin A}{\cos A}$ $\qquad \cot A = \frac{\cos A}{\sin A}$

Pythagorean Identities:

$$\sin^2 A + \cos^2 A = 1 \qquad \tan^2 A + 1 = \sec^2 A \qquad 1 + \cot^2 A = \csc^2 A$$

Sum and Difference Identities:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B \qquad \sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B \qquad \cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Double Angle Identities:

$$\sin(2A) = 2\sin A \cos A \qquad \tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

$$\cos(2A) = \cos^2 A - \sin^2 A \qquad \cos(2A) = 2\cos^2 A - 1 \qquad \cos(2A) = 1 - 2\sin^2 A$$

Half Angle Identities:

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}} \qquad \cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}} \qquad \tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

Polar Formulas:

$$x^2 + y^2 = r^2 \qquad x = r\cos\theta \qquad y = r\sin\theta \qquad \tan^{-1}\frac{y}{x} = \theta \quad x > 0, \qquad \tan^{-1}\frac{y}{x} = \theta + \pi \quad x < 0$$

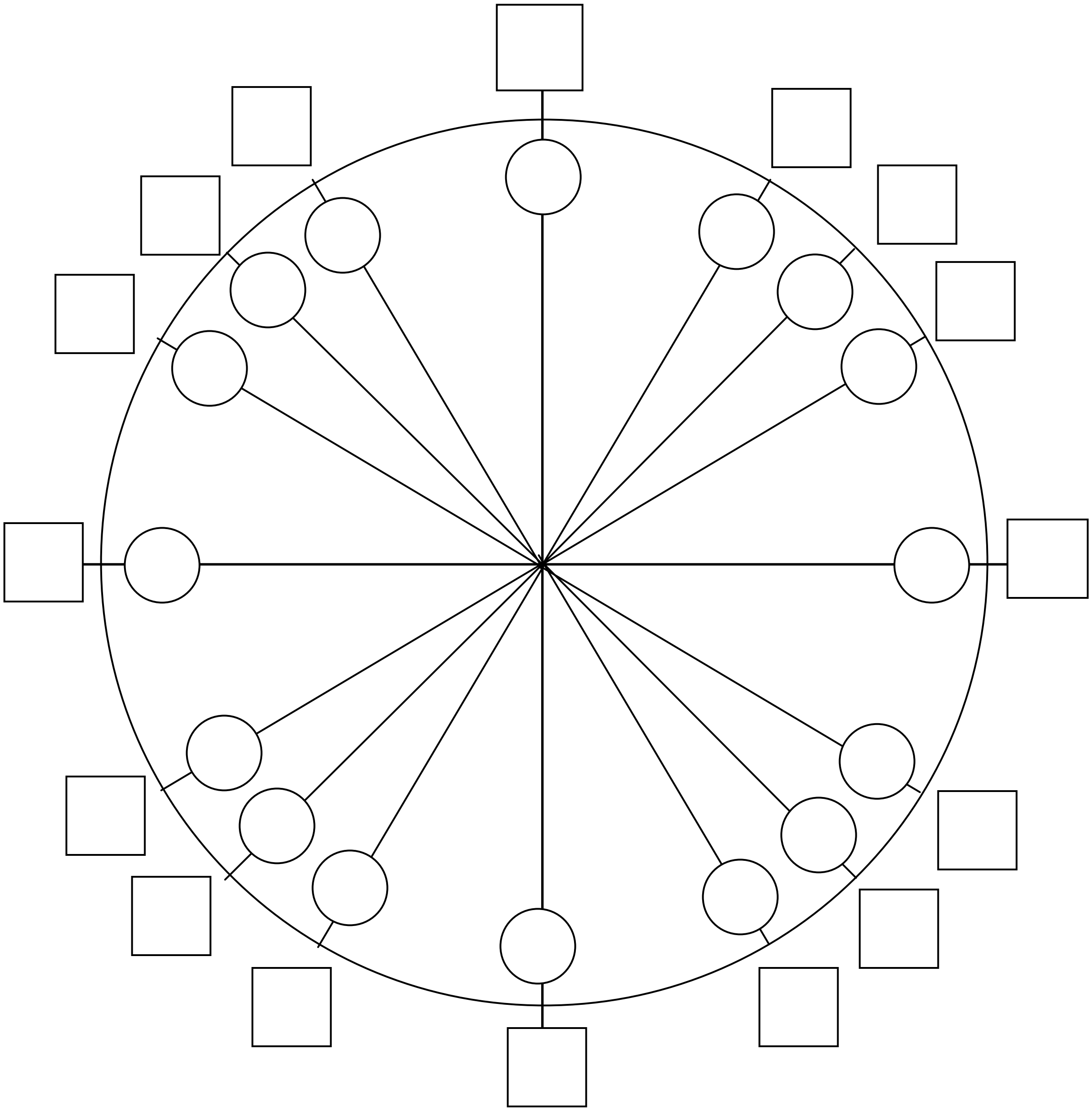
Geometric Formulas:

Area of a trapezoid: $A = \frac{1}{2}h\ \ b_1 + b_2$ $\qquad$ Area of a triangle: $A = \frac{1}{2}bh$

Area of an equilateral triangle: $A = \frac{\sqrt{3}}{4}\ s^2$

Area if a circle: $A = \pi r^2$ $\qquad$ Circumference of a circle: $C = 2\pi r$ or $C = d\pi$

Unit Circle – Degrees and Radians



Place degree measures in the circles. $\tan\theta$ = ________________

Place radian measure in the squares. $\cot\theta$ = ________________

Place $(\cos\theta, \sin\theta)$ in parenthesis outside the square. $\csc\theta$ = ________________

Place $\tan\theta$ outside the parenthesis. $\sec\theta$ = ________________

SKILLS NEEDED FOR CALCULUS

I. Algebra:

*A. Exponents (operations with integer, fractional, and negative exponents)
*B. Factoring (GCF, trinomials, difference of squares and cubes, sum of cubes, grouping)
C. Rationalizing (numerator and denominator)
*D. Simplifying rational expressions
*E. Solving algebraic equations and inequalities (linear, quadratic, higher order using synthetic division, rational, radical, and absolute value equations)
F. Simultaneous equations

II. Graphing and Functions

*A. Lines (intercepts, slopes, write equations using point-slope and slope intercept, parallel, perpendicular, distance and midpoint formulas)
B. Conic Sections (circle, parabola, ellipse, and hyperbola)
*C. Functions (definition, notation, domain, range, inverse, composition)
*D. Basic shapes and transformations of the following functions (absolute value, rational, root, higher order curves, log, ln, exponential, trigonometric. piece-wise, inverse functions)
E. Tests for symmetry: odd, even

III. Geometry

A. Pythagorean Theorem
B. Area Formulas (Circle, polygons, surface area of solids)
C. Volume formulas
D. Similar Triangles

**** IV. Logarithmic and Exponential Functions***

*A. Simplify Expressions (Use laws of logarithms and exponents)
*B. Solve exponential and logarithmic equations (include ln as well as log)
*C. Sketch graphs
*D. Inverses

**** V. Trigonometry***

**A. Unit Circle (definition of functions, angles in radians and degrees)
B. Use of Pythagorean Identities and formulas to simplify expressions and prove identities
*C. Solve equations
*D. Inverse Trigonometric functions
E. Right triangle trigonometry
*F. Graphs

* A solid working foundation in these areas is very important.

Calculus Prerequisite Problems

Work the following problems on your own paper. Show all necessary work.

I. Algebra

A. Exponents: 1) $\dfrac{{8x^3yz}^{\frac{1}{3}}\;{2x}^{3}}{4x^{\frac{1}{3}}\;{yz^{\frac{2}{3}}}^{-1}}$

B. Factor Completely:

2) $9x^2 + 3x - 3xy - y$ (use grouping)

3) $64x^6 - 1$ *Hint: Factor as difference of squares first, then as the sum and difference of cubes second.*

4) $42x^4 + 35x^2 - 28$

5) $15x^{5/2} - 2x^{3/2} - 24x^{1/2}$ Hint: Factor GCF $x^{1/2}$ first.

6) $x^{-1} - 3x^{-2} + 2x^{-3}$ Hint: Factor out GCF x^{-3} first.

C. Rationalize denominator / numerator:

7) $\dfrac{3-x}{1-\sqrt{x-2}}$

8) $\dfrac{\sqrt{x+1}+1}{x}$

D. Simplify the rational expression:

9) $\dfrac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

E. Solve algebraic equations and inequalities

10. – 11. *Use synthetic division to help factor the following, state all factors and roots.*

10) $p(x) = x^3 + 4x^2 + x - 6$

11) $p(x) = 6x^3 - 17x^2 - 16x + 7$

12) *Explain* why $\dfrac{3}{2}$ cannot be a root of $f(x) = 4x^5 + cx^3 - dx + 5$, where c and d are integers. (hint: You can look at the possible rational roots.)

13) *Explain* why $f(x) = x^4 + 7x^2 + x - 5$ must have a root in the interval [0, 1], ($0 \le x \le 1$) Check the graph and use signs of f(0) and f(1) to justify you're answer.

Solve: You may use your graphic calculator to check solutions.

14) $(x + 3)^2 > 4$

15) $\dfrac{x+5}{x-3} \le 0$

16) $3x^3 - 14x^2 - 5x \le 0$ (Factor first)

17) $x < \frac{1}{x}$ 18) $\frac{x^2 - 9}{x + 1} \geq 0$ 19) $\frac{1}{x - 1} + \frac{4}{x - 6} > 0$

20) $x^2 < 4$ 21) $|2x + 1| < \frac{1}{4}$

F. Solve the system. Solve the system algebraically and then check the solution by graphing each function and using your calculator to find the points of intersection.

22) $x - y + 1 = 0$
$y - x^2 = -5$

23) $x^2 - 4x + 3 = y$
$-x^2 + 6x - 9 = y$

II. Graphing and Functions:

A. Linear graphs: *Write the equation of the line described below.*

24) Passes through the point (2, -1) and has slope $-\frac{1}{3}$.

25) Passes through the point (4, - 3) and is perpendicular to $3x + 2y = 4$.

26) Passes through (-1, - 2) and is parallel to $y = \frac{3}{5}x - 1$.

B. Conic Sections: *Write the equation in standard form and identify the conic.*

27) $x = 4y^2 + 8y - 3$ 28) $4x^2 - 16x + 3y^2 + 24y + 52 = 0$

C. Functions: *Find the domain and range of the following.*

Note: domain restrictions - denominator ≠ 0, argument of a log or ln > 0,
radicand of even index must be ≥ 0
range restrictions- reasoning, if all else fails, use graphing calculator

29) $y = \frac{3}{x - 2}$ 30) $y = \log(x - 3)$ 31) $y = x^4 + x^2 + 2$

32) $y = \sqrt{2x - 3}$ 33) $y = |x - 5|$ 34) domain only: $y = \frac{\sqrt{x+1}}{x^2 - 1}$

35) Given f(x) below, graph over the domain [-3, 3], what is the range?

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 1 & \text{if } -1 \leq x < 0 \\ x - 2 & \text{if } x < -1 \end{cases}$$

Find the composition /inverses as indicated below.

Let **$f(x) = x^2 + 3x - 2$** **$g(x) = 4x - 3$** **$h(x) = \ln x$** **$w(x) = \sqrt{x - 4}$**

36) $g^{-1}(x)$ 37) $h^{-1}(x)$ 38) $w^{-1}(x)$, for $x \geq 4$

39) f(g(x)) 40) h(g(f(1)))

41) Does $y = 3x^2 - 9$ have an inverse function? Explain your answer.

Let **$f(x) = 2x$, $g(x) = -x$, and $h(x) = 4$,** find

42) $(f \circ g)(x)$ 43) $(f \circ g \circ h)(x)$

44) Let $s(x) = \sqrt{4 - x}$ and $t(x) = x^2$, find the domain and range of $(s \circ t)(x)$.

D. Basic Shapes of Curves:

Sketch the graphs. You may use your graphing calculator to verify your graph, but you should be able to graph the following by knowledge of the shape of the curve, by plotting a few points, and by your knowledge of transformations.

45) $y = \sqrt{x}$ 46) $y = \ln x$ 47) $y = \frac{1}{x}$ 48) $y = | x - 2 |$

49) $y = \frac{1}{x - 2}$ 50) $y = \frac{x}{x^2 - 4}$ 51) $y = 2^{-x}$ 52) $y = 3 \sin 2 (x - \frac{\pi}{6})$

53) $$f(x) = \begin{cases} \sqrt{25 - x^2} & \text{if } x < 0 \\ \dfrac{x^2 - 25}{x - 5} & \text{if } x \geq 0, x \neq 5 \\ 0 & \text{if } x = 5 \end{cases}$$

E. Even, Odd, Tests for Symmetry:

Id*entify as odd, even, or neither and justify your answer.* *To justify your answer you must show substitution using -x ! It is not enough to simply check a number*.

Even: f (x) = f (-x) Odd: f (-x) = - f (x)

54) $f(x) = x^3 + 3x$ 55) $f(x) = x^4 - 6x^2 + 3$ 56) $f(x) = \frac{x^3 - x}{x^2}$

57) $f(x) = \sin 2x$ 58) $f(x) = x^2 + x$ 59) $f(x) = x(x^2 - 1)$

60) $f(x) = \frac{1 + |x|}{x^2}$

61) What type of function (even or odd) results from the product of two

even functions? odd functions?

Test for symmetry. Show substitution with variables to justify your answer.

Symmetric to y axis: replace x with - x and relation remains the same.
Symmetric to x axis: replace y with - y and relation remains the same.
Origin symmetry: replace x with - x, y with - y and the relation is equivalent.

62) $y = x^4 + x^2$ 63) $y = \sin(x)$ 64) $y = \cos(x)$

65) $x = y^2 + 1$ 66) $y = \frac{|x|}{x^2 + 1}$

IV LOGARITHMIC AND EXPONENTIAL FUNCTIONS

A. Simplify Expressions:

67) $\log_4 \frac{1}{16}$ 68) $3\log_3 3 - \frac{3}{4}\log_3 81 + \frac{1}{3}\log_3 \frac{1}{27}$ 69) $\log_9 27$

70) $\log_{125} \frac{1}{5}$ 71) $\log_w w^{45}$ 72) $\ln e$ 73) $\ln 1$ 74) $\ln e^2$

B. Solve equations:

75) $\log_6(x + 3) + \log_6(x + 4) = 1$ 76) $\log x^2 - \log 100 = \log 1$ 77) $3^{x+1} = 15$

V TRIGONOMETRY

A. Unit Circle: Know the unit circle – radian and degree measure.
Be prepared for a quiz.

78) State the domain, range and fundamental period for each function.

a) $y = \sin x$ b) $y = \cos x$ c) $y = \tan x$

B. Identities:

Simplify: 79) $\dfrac{(\tan^2 x)(\csc^2 x) - 1}{(\csc x)(\tan^2 x)(\sin x)}$ 80) $1 - \cos^2 x$ 81) $\sec^2 x - \tan^2 x$

82) Verify : $(1 - \sin^2 x)(1 + \tan^2 x) = 1$

C. Solve the Equations

83) $\cos^2 x = \cos x + 2, \quad 0 \le x \le 2\pi$ 84) $2\sin(2x) = \sqrt{3}, \quad 0 \le x \le 2\pi$

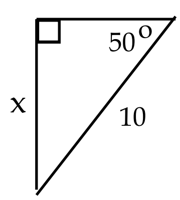
85) $\cos^2 x + \sin x + 1 = 0, \quad 0 \le x \le 2\pi$

D. Inverse Trig Functions: ***Note:*** $\mathit{Sin}^{-1} x = \mathit{Arcsin}\ x$

86) Arcsin1 87) $\operatorname{Arcsin}\left(-\dfrac{\sqrt{2}}{2}\right)$ 88) $\operatorname{Arccos}\left(\dfrac{\sqrt{3}}{2}\right)$ 89) $\sin\left(\operatorname{Arccos}\left(\dfrac{\sqrt{3}}{2}\right)\right)$

90. State domain and range for: Arcsin(x) , Arccos (x), Arctan (x)

E. Right Triangle Trig: Find the value of x. (Note: Degree measure!)

91.


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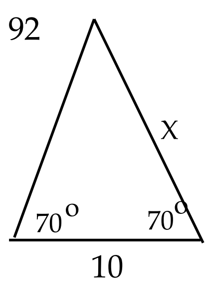

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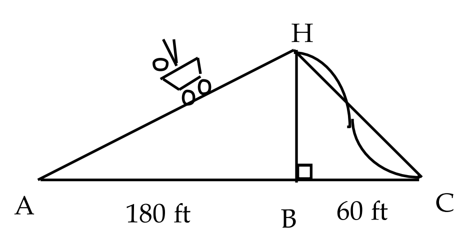

93) The roller coaster car shown in the diagram above takes 23.5 sec. to go up the 23 degree incline segment AH and only 2.8 seconds to go down the drop from H to C. The car covers horizontal distances of 180 feet on the incline and 60 feet on the drop. Decimals in answer may vary.

a. How high is the roller coaster above point B?
b. Find the distances AH and HC.
c. How fast (in ft/sec) does the car go up the incline?
d. What is the approximate average speed of the car as it goes down the drop?
e. Assume the car travels along HC. Is your approximate answer too big or too small?

(Advanced Mathematics, Richard G. Brown, Houghton Mifflin,1994, pg 336)

F. Graphs: Identify the amplitude, period, horizontal, and vertical shifts of these functions.

94) $y = -2\sin(2x)$ 95) $y = -\pi\cos\frac{\pi}{2}x + \pi$

G. Be able to do the following on your graphing calculator:

Be familiar with the CALC commands; value, root, minimum, maximum, intersect.

You may need to zoom in on areas of your graph to find the information.
Answers should be accurate to 3 decimal places. Sketch the graph.

96. – 99. Given the following function $f(x) = 2x^4 - 11x^3 - x^2 + 30x$.

96. Find all roots.

Note: Window x min: -10 x max: 10 scale 1
y min: - 100 y max: 60 scale 0

97. Find all local maxima.

98. Find all local minima.

A local maximum or local minimum is a point on the graph where there is a highest or lowest point within an interval such as the vertex of a parabola.

99. Find the following values: f (-1), f (2), f (0), f (.125)

100. Graph the following two functions and find their points of intersection using the intersect command on your calculator.

$y = x^3 + 5x^2 - 7x + 2$ and $y = .2x^2 + 10$ Window: x min : -10 x max: 10 scale 1
y min: - 10 y max: 50 scale 0